

# Self-organized network evolution coupled to extremal dynamics

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The interplay between topology and dynamics in complex networks is a fundamental but widely unexplored problem. Here, we study this phenomenon on a prototype model in which the network is shaped by a dynamical variable. We couple the dynamics of the Bak–Sneppen evolution model with the rules of the so-called fitness network model for establishing the topology of a network; each vertex is assigned a 'fitness', and the vertex with minimum fitness and its neighbours are updated in each iteration. At the same time, the links between the updated vertices and all other vertices are drawn anew with a fitness-dependent connection probability. We show analytically and numerically that the system self-organizes to a non-trivial state that differs from what is obtained when the two processes are decoupled. A power-law decay of dynamical and topological quantities above a threshold emerges spontaneously, as well as a feedback between different dynamical regimes and the underlying correlation and percolation properties of the network.

The properties of dynamical processes defined on complex networks exhibit a strong dependence on the topology<sup>1–4</sup>. On the other hand, there is growing empirical evidence<sup>5–7</sup> that many networks are in turn shaped by some variable associated with each vertex, an aspect captured by the 'fitness' or 'hidden-variable' model<sup>8,9</sup>. Until now, these two facets of the same problem have been treated as separate, by considering on one hand dynamical processes on static networks<sup>1,4</sup>, and on the other hand network formation mechanisms driven by quenched variables<sup>8–12</sup>. This may perhaps be justified for short timescales. However, in the long-term evolution, it is crucial to understand the effects that these mechanisms have on each other, without *ad hoc* specifications of any fixed structure either in the topology or in the dynamical variables. Remarkably, the interplay of dynamics and topology can drive the network to a self-organized state that cannot be inferred by studying the two evolutionary processes as decoupled.

Here, we explore explicitly the possibility that the network supports a dynamical process which in turn shapes its topology, with a continuous feedback between dynamics and structure. Models where both dynamical and topological properties are continuously updated have been considered<sup>13–18</sup>. In these cases, however, the rewiring of links is not completely driven by the dynamical variables. In contrast, our main interest here is the description of a self-organized process where the dynamical variable fully acts also as the 'hidden variable' shaping network topology explicitly, as in the fitness model. Owing to the increased complexity of the problem, we choose the simplest possible dynamical rule for the hidden variable. We focus on the extremal dynamics defined in the Bak–Sneppen model<sup>19</sup>, a traditional model of self-organized criticality<sup>20</sup> (SOC) inspired by

biological evolution. As the outcomes of this model on a wide range of fitness-independent networks are well known<sup>19,21–26</sup>, it is straightforward to understand what the novel effects originating uniquely by the interplay with the fitness-driven topological evolution we consider here are.

## CHARACTERIZATION OF THE MODEL

In the traditional Bak–Sneppen model<sup>19</sup> defined on a generic graph<sup>21–26</sup>, each of the  $N$  vertices is regarded as a biological species having a fitness value  $x_i$ , initially drawn from a uniform distribution between 0 and 1. At each time step, the species with the lowest fitness and all of its neighbours undergo a mutation, and their fitness values are drawn anew from the same uniform distribution. The process is iterated, and eventually the system reaches a stationary state, characterized by a step-like fitness distribution, uniform above a threshold  $\tau$ . This behaviour is observed on regular lattices<sup>19,21</sup>, random graphs<sup>22</sup>, small-world<sup>23</sup> and scale-free<sup>24–26</sup> networks, the only dependence on the particular topology being in the value of  $\tau$  (refs 19,21–26). In particular,  $\tau$  vanishes for scale-free degree distributions with a diverging second moment<sup>24–26</sup>.

Here, we couple this dynamical rule with the fitness model assumption<sup>8</sup> that the network is formed by drawing a link between any two vertices  $i$  and  $j$  with fitness-dependent probability  $f(x_i, x_j)$ , thus introducing an intrinsic feedback between dynamics and topology. In this way, whenever the fitness  $x_i$  of a species  $i$  is updated to  $x'_i$ , the links from  $i$  to all of the other vertices  $j$  are drawn anew with probability  $f(x'_i, x_j)$ . Besides the updates described above, further and arbitrary link updating events could

also be defined. In other words, in addition to the ‘natural’ link update occurring between a mutating species and all other species, other link updates may happen between any two vertices  $i$  and  $j$  at generic and arbitrarily distributed time steps. When this occurs, the link between  $i$  and  $j$  is updated and drawn anew with probability  $f(x_i, x_j)$ , where  $x_i$  and  $x_j$  are the current fitness values of  $i$  and  $j$ , even if the latter are not involved in a mutation event. Remarkably, it is possible to show (see the Supplementary Information) that the introduction of link updating events leaves the system in the same stationary state as if they were absent. Therefore, our model is very general in this respect, and allows for rearrangements of ecological interactions on shorter timescales than those generated by mutations. In particular, the stationary state is the same if the whole network is updated at each time step. In this case, storing the information on the adjacency matrix among species is unnecessary, and we shall exploit this property to achieve fast and very large numerical simulations of the model.

As we show below, the coupling between structure and dynamics leads to unexpected results that cannot be traced back to any of the two processes taken as separate. Moreover, another important advantage is that the main limitations of the two models disappear when they are coupled together. A fundamental problem in the Bak–Sneppen model on static graphs is that, after a mutation, the new species always inherits exactly all of the links of the previous one. This is hard to justify, as it is precisely the structure of ecological connections among species that is believed to be both the origin and the outcome of macroevolution<sup>13</sup>. Here, the fitness-driven link updating overcomes this problem. Similarly, the static fitness model requires the specification of an *ad hoc* fitness distribution that never changes. In contrast, here the fitness distribution self-organizes spontaneously to a stationary probability density, removing the need of arbitrary specifications. As we discuss below, a proper interpretation of the fitness also allows us to remove the remaining arbitrariness in the choice of  $f(x_i, x_j)$ . However, to keep our approach as general as possible, we first study the model analytically for a generic form of  $f$ , and focus on particular choices only later.

The analytical solution of the model for an arbitrary linking function  $f(x, y)$  can be obtained by focusing on the master equation for the fitness distribution  $\rho(x)$  at the stationary state (see the Supplementary Information). We find that the analytical expression for  $\rho(x)$  is

$$\rho(x) = \begin{cases} (\tau N)^{-1} & x < \tau \\ \frac{1}{N \int_0^\tau f(x, m) dm} & x > \tau, \end{cases} \quad (1)$$

where  $\tau$  is a threshold value determined through the normalization condition  $\int_0^1 \rho(x) dx$ , which reads

$$\int_\tau^1 \frac{dx}{\int_0^\tau f(x, m) dm} = N - 1. \quad (2)$$

In the infinite size limit  $N \rightarrow \infty$ , the distribution  $q(m)$  of the minimum fitness value  $m \equiv x_{\min}$  is uniform between 0 and  $\tau$ , whereas all other values (except possibly a vanishing fraction) are above  $\tau$  (see the Supplementary Information). In other words,  $q(m) = \Theta(\tau - m) / \tau$ , where  $\Theta$  is the usual step function ( $\Theta(x) = 0$  if  $x < 0$  and  $\Theta(x) = 1$  if  $x > 0$ ). This characterizes the stationary state completely. Once  $\rho(x)$  is known, all of the expected topological quantities can be determined as in the static fitness model<sup>8,11,12</sup>. For instance, the average degree of a vertex with fitness  $x$  is given by

$$k(x) = N \int_0^1 f(x, y) \rho(y) dy \quad (3)$$

and the inverse function  $x(k)$  can be used to obtain the cumulative degree distribution as

$$P_{>}(k) \equiv \int_k^{k(1)} P(k') dk' = \rho_{>}[x(k)], \quad (4)$$

where  $\rho_{>}(x) \equiv \int_x^1 \rho(x') dx'$  is the cumulative fitness distribution. Note that if  $\tau$  is non-zero, the fitness distribution preserves the discontinuous behaviour exhibited on static networks<sup>19,21–26</sup>. However, here we find the novel feature that  $\rho(x)$  is in general not uniform for  $x > \tau$ . This unexpected result holds for any non-trivial choice of  $f(x, y)$ , and hence for any topology. Therefore, the effect is not due to the topology itself, but to the interplay between the topological evolution and the dynamical process entangled with it. Remarkably, this feedback alone determines the self-organization of the system from a random structure to a complex network with non-trivial dynamical and topological properties.

### FITNESS-INDEPENDENT RANDOM GRAPHS

The above analytical solution holds for any form of  $f(x, y)$ . Now we consider possible choices of this function. First note that the null choice is  $f(x, y) = p$ , the network being a random graph. It is nonetheless an instructive simple case, and we briefly discuss it. Moreover, this choice is asymptotically equivalent to the random neighbour variant<sup>22</sup> of the Bak–Sneppen model, the average degree of each vertex being  $d = p(N - 1) \approx pN$  (we drop terms of order  $1/N$  from now on). Our analytical results read

$$\rho(x) = \begin{cases} (\tau N)^{-1} & x < \tau \\ (p\tau N)^{-1} & x > \tau \end{cases} \quad (5)$$

and, depending on how  $p$  scales with  $N$ , equation (2) implies

$$\tau = \frac{1}{1 + pN} \rightarrow \begin{cases} 1 & pN \rightarrow 0 \\ (1 + d)^{-1} & pN = d \\ 0 & pN \rightarrow \infty. \end{cases} \quad (6)$$

We note that these three dynamical regimes are tightly related to an underlying topological phase transition. As  $p$  decreases, the whole system splits up into a number of smaller subsets or clusters. This process exhibits a critical behaviour near the threshold  $p_c \approx 1/N$  (refs 3,4). Below  $p_c$ , each node is isolated or linked to a small number of peers. Above  $p_c$ , a large giant component emerges including a number of nodes of order  $O(N)$ , whose fraction tends to 1 as  $p \rightarrow 1$ . This explains the dynamical regimes in equation (6). If  $pN \rightarrow \infty$  (dense regime), then  $\tau \rightarrow 0$  and  $\rho(x)$  is uniform between 0 and 1 as in the initial state, as an infinite number  $\langle k_{\min} \rangle = pN$  of fitnesses is continuously updated as on a complete graph. In this case, the step-like behaviour is destroyed. If  $pN = d$  with finite  $d > 1$  (sparse regime), then  $\tau$  remains finite as  $N \rightarrow \infty$ , and this is the case considered in ref. 22 that we recover correctly. Finally, if  $p$  falls faster than  $1/N$ , the graph is below the percolation threshold (subcritical regime): the updates cannot propagate and  $\tau \rightarrow 1$ , as for  $N$  isolated vertices (only the minimum is continuously updated, which after many time steps results in pushing all fitness values, except the newly replaced one, towards 1). Therefore, the dynamical transition is rooted in an underlying topological phase transition. This previously unrecognized property is fundamental and, as we show below, is also general.

### FITNESS-DEPENDENT COMPLEX NETWORKS

A non-trivial form of  $f(x, y)$  must be chosen carefully. On static or fitness-independent networks,  $x_i$  is usually interpreted as the fitness

barrier against further mutation, and the links are interpreted as feeding relations<sup>19</sup>. However, once the topology depends on  $x$  these two interpretations are difficult to concile. The coupling we have introduced requires consistent interpretations of  $x$  and of the links. In addition, the form of  $f(x, y)$  must be consistent with the feature that the updates of  $x$  propagate through the network determined by it. This instructive aspect must characterize any model with coupled topology and dynamics, and reduces significantly the arbitrariness introduced in the static case. Here, we suggest that the simplest self-consistent interpretation is the following. As there is no external world in the model, the environment experienced by a species is simply the set of its ecological interactions. Now let  $x_i$  represent the fitness (rather than the barrier) of  $i$ , and let a link between two species mean ‘being fit to coexist with each other’ (that is, it represents an undirected, non-feeding interaction beneficial to both). The more a species is connected to other species, the more it is fitted to the environment. This picture is self-consistent provided that the larger  $x$  and  $y$  are, the larger  $f(x, y)$  is. Following the results of refs 27,28, the simplest unbiased<sup>28</sup> choice for such a function is

$$f(x, y) = \frac{zxy}{1 + zxy}, \quad (7)$$

where  $z$  is a positive parameter controlling the number of links. This choice generates a network with a nonrandom, fitness-dependent expected degree sequence<sup>27,28</sup>, which in this case is not known *a priori* and will be determined by the fitness distribution at the stationary state. All other higher-order properties are completely random, except for the structural correlations induced by the degree sequence<sup>27,28</sup>. It therefore represents the fitness-dependent version of the so-called configuration model<sup>4,29</sup>. As we show later, structural correlations have an important impact on the dynamics. With the above choice,  $\rho(x)$  can be directly computed analytically through equation (1). However, we write it in a different form, which is equivalent when  $N \rightarrow \infty$ , to also solve more complicated integrals involving it later. We use  $\langle f(x, m) \rangle \approx f(x, \langle m \rangle)$ , where the angular brackets denote an average over the distribution  $q(m)$  of the minimum fitness, that is,  $\tau^{-1} \int_0^\tau f(x, m) dm \approx (zx\tau/2)/(1 + zx\tau/2)$ . As we show below, when  $N \rightarrow \infty$  this approximated expression becomes exact. Then equation (1) yields

$$\rho(x) = \begin{cases} (\tau N)^{-1} & x < \tau \\ (\tau N)^{-1} + 2/(zN\tau^2 x) & x > \tau, \end{cases} \quad (8)$$

where  $\tau$  is the solution of equation (2), which reads

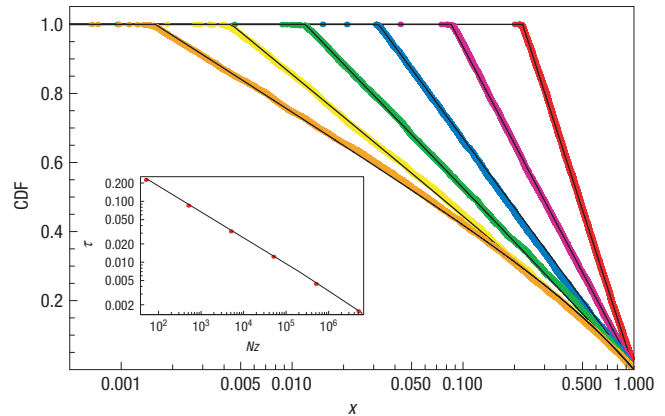
$$\frac{1}{\tau} + \frac{1}{z\tau^2} \log \frac{1}{\tau^2} = N.$$

If  $z$  remains finite as  $N \rightarrow \infty$ , or in other words if  $zN \rightarrow \infty$ , then the trivial solution is  $\tau \rightarrow 0$ . On the other hand, we find  $\tau \neq 0$  if  $zN$  remains finite as  $N \rightarrow \infty$ . To obtain the value of  $\tau$  in this case, note that for a non-zero solution the term  $1/\tau$  in the above expression is finite and negligible for  $N$  large enough. Multiplying both sides by  $z$  yields

$$\frac{1}{\tau^2} \log \frac{1}{\tau^2} = zN,$$

whose solution is  $\tau = \sqrt{\phi(zN)/zN}$ , where  $\phi(x)$  is the so-called ProductLog function, defined as the solution of  $\phi e^\phi = x$ . Putting these results together, we have

$$\tau = \sqrt{\frac{\phi(zN)}{zN}} \rightarrow \begin{cases} 1 & zN \rightarrow 0 \\ \sqrt{\phi(d)/d} & zN = d \\ 0 & zN \rightarrow \infty. \end{cases}$$



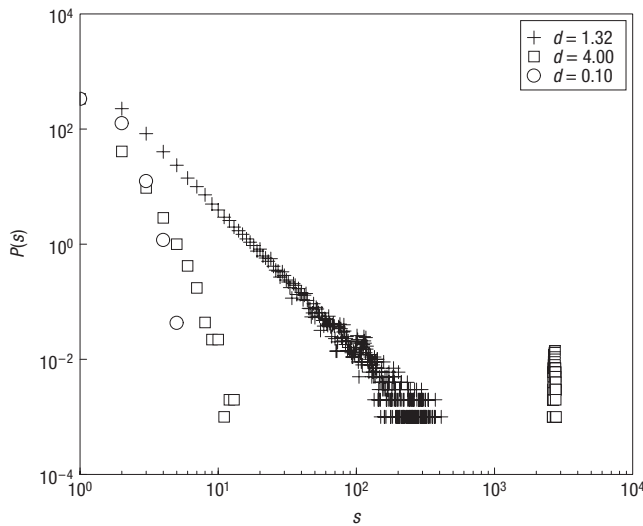
**Figure 1** Stationary fitness distribution and threshold. CDF  $\rho_>(x)$  in log-linear axes. From right to left,  $z = 0.01$ ,  $z = 0.1$ ,  $z = 1$ ,  $z = 10$ ,  $z = 100$ ,  $z = 1,000$  ( $N = 5,000$ ). Inset: Log-log plot of  $\tau(zN)$ . Solid lines: theoretical curves, points: simulation results.

As for random graphs, we find a marked transition as the scaling of  $z$  changes from  $N^{-1}$  to more rapidly decaying. This suggests an analogous underlying percolation transition. As we show below, this is indeed the case. We can therefore still refer to the subcritical, sparse and dense regimes. Note that as  $N \rightarrow \infty$ , we have  $f(x, y) = zxy$  in the sparse and subcritical regimes because  $zxy < z \ll 1$ , which implies that we can neglect  $zxy$  in the denominator of equation (7). Therefore, the expression  $\langle f(x, m) \rangle = f(x, \langle m \rangle)$  is exact. On the other hand, in the dense regime, we have  $\tau \rightarrow 0$ , which again implies the same expression because  $q(m)$  becomes the Dirac delta function  $\delta(m)$ . Therefore, our use of the above expression turns out to be exact in all regimes for  $N \rightarrow \infty$ .

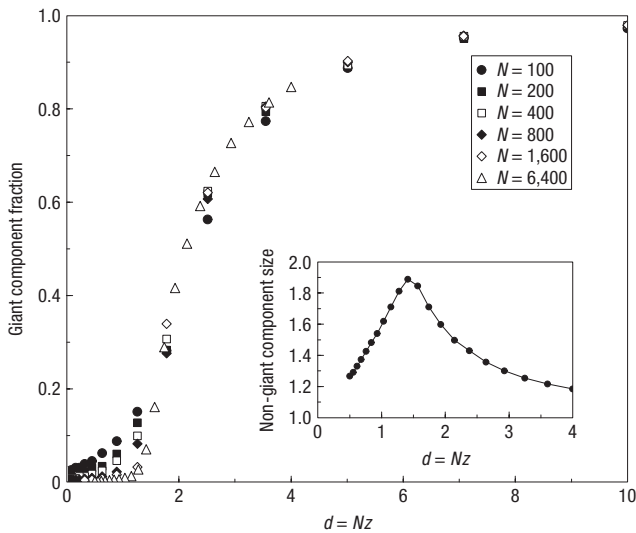
## RESULTS AND DISCUSSION

Figure 1 shows the cumulative density function (CDF) of the fitness  $\rho_>(x)$ , and the inset shows a plot of  $\tau(zN)$ . The theoretical results are in excellent agreement with numerical simulations. As predicted by equation (8),  $\rho(x)$  is the superposition of a uniform distribution and a power-law with exponent  $-1$ . For  $z \ll 1$ , we have  $f(x, y) \approx zxy$  and  $\rho(x) \propto x^{-1}$  for  $x > \tau$ . This purely power-law behaviour, that becomes exact in the sparse regime  $z = d/N$  for  $N \rightarrow \infty$ , results in a logarithmic CDF looking like a straight line in log-linear axes. Note that, despite the value of the exponent, the presence of a non-zero lower threshold ensures that  $\rho(x)$  is normalizable. This mechanism may provide a natural explanation for the onset of Pareto distributions with a finite minimum value in real systems. In contrast, for large  $z$  the uniform part is non-vanishing and  $\rho(x)$  deviates from the purely power-law behaviour. The decay of  $\rho(x)$  for  $x > \tau$  is a completely novel outcome of the extremal dynamics due to the feedback with the topology: now the fittest species at a given time is also the most likely to be connected to the least fit species and to mutate at the following time step. Being more connected also means being more subject to changes. This enriches the coexistence patterns exhibited on static networks.

We now check the conjectured percolation transition. For different system sizes, we find that the cluster size distribution  $P(s)$  exhibits power-law tails when the control parameter  $d \equiv zN$  approaches a critical value  $d_c = 1.32 \pm 0.05$  (corresponding to  $z_c = d_c/N$ ), suggesting the onset of a second-order percolation-like phase transition. As shown in Fig. 2,  $P(s) \propto s^{-\gamma}$  where  $\gamma$  is the



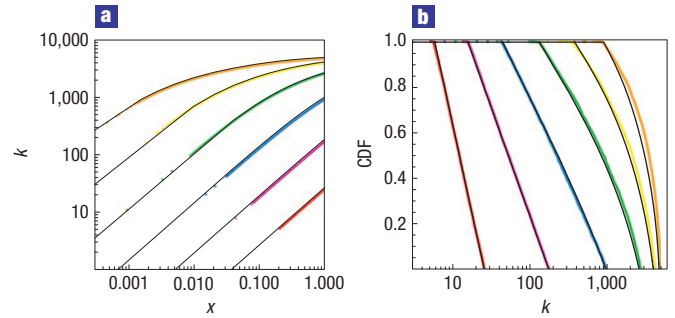
**Figure 2 Cluster size distribution.** Far from the critical threshold ( $d = 0.10$  and  $d = 4.00$ ),  $P(s)$  is well peaked. At  $d_c = 1.32$ ,  $P(s) \propto s^{-\gamma}$  with  $\gamma = 2.45 \pm 0.05$ . Here,  $N = 3,200$ .



**Figure 3 Behaviour at the percolation threshold.** The fraction of nodes in the giant component for different network sizes as a function of  $d$ . Inset: The non-giant component average size as a function of  $d$  for  $N = 6,400$ .

exponent of the power law and takes the value  $\gamma = 2.45 \pm 0.05$  at the phase transition. Figure 3 shows that the average fraction of nodes in the largest component remains negligible for  $d < d_c$ , whereas it takes increasing finite values above  $d_c$ . As a further check, following the method adopted in ref. 30, we have plotted the average size fraction of non-giant components, which diverges (in the infinite volume limit) when  $P(s)$  decays algebraically as reported in the inset of Fig. 3.

Although one of the most studied properties of the Bak-Sneppen model on regular lattices is the statistics of avalanches characterizing the SOC behaviour<sup>19</sup>, we do not consider it here. This is because, as shown in ref. 31, when considering long-range<sup>22</sup> instead of nearest-neighbour connections, it can lead to a wrong



**Figure 4 Fitness dependence and cumulative distribution of the degrees.** **a**,  $k(x)$  ( $N = 5,000$ ; from right to left,  $z = 0.01$ ,  $z = 0.1$ ,  $z = 1$ ,  $z = 10$ ,  $z = 100$ ,  $z = 1,000$ ). **b**,  $P_>(k)$  (same parameter values, inverse order from left to right). Solid lines: theoretical curves, points: simulation results.

assessment of the SOC state, which is put into question by the absence of spatial correlations even in the case that avalanches are power-law distributed. Rather, we further characterize the topology at the stationary state by considering the degree distribution  $P(k)$  and the degree correlations. Using equation (3), we find that the average degree  $k(x)$  of a vertex with fitness  $x$  is

$$k(x) = \frac{2}{z\tau^2} \ln \frac{1+zx}{1+z\tau x} + \frac{zx - \ln(1+zx)}{z\tau x}. \quad (9)$$

Similarly, through equation (4) we can determine the analytical expression for the cumulative degree distribution  $P_>(k)$ . As shown in Fig. 4,  $k(x)$  is linear for small  $z$  because  $f(x, z) \approx zxy$ , whereas for large  $z$  it saturates to the maximum value  $k_{\max} = k(1)$ . This implies that in the sparse regime,  $P(k)$  mimics  $\rho(x)$  and is characterized by the threshold value  $k(\tau)$  and by a power-law decay  $P(k) \propto k^{-1}$  above it (see Fig. 4). Note that here  $\tau$  remains finite even if  $P(k) \propto k^{-\gamma}$  with  $\gamma < 3$ , in striking contrast with that obtained on static scale-free networks<sup>24–26</sup>. In contrast, for large  $z$  the saturation  $k \rightarrow k_{\max}$  translates into a cutoff that makes  $P(k)$  deviate from the pure power-law behaviour for  $k > k(\tau)$ . As shown in refs 5,27, this saturation determines anticorrelation between the degrees of neighbouring vertices (disassortativity) and a hierarchy of degree-dependent clustering coefficients as observed in many real-world networks (this is not shown here for brevity). As  $N \rightarrow \infty$ , these correlations vanish in the sparse regime ( $\tau > 0$ ), whereas they survive in the dense regime ( $\tau \rightarrow 0$ ). Structural correlations and a non-zero threshold  $\tau$  are then mutually excluding in this model, which is another interesting effect of the feedback we have introduced. Finally, we note that with a different choice of  $f$ , we can have any exponent for the power-law distribution of fitnesses, and therefore for the degree distribution as well. This makes our model completely flexible to reproduce any desired topological property.

Our results represent a first step into the unexplored domain of systems with generic self-organized coupling between dynamics and topology. A huge class of such processes needs to be studied in the future, to further understand the unexpected effects of this coupling.

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#### Author contributions

D.G. developed the theory and carried out computer simulations. A.C. carried out computer simulations. G.C. planned the study and developed the theory.

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